Feedback effects on the pairing interaction in color superconductors near the transition temperature

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(Dated: February 2, 2008)

We examine the role that the gap dependence of the pairing interaction plays in the gap equation for a weakly coupled uniform superfluid of three-flavor massless quarks near the transition temperature T_c. We find that the feedback effects on Landau-damped transverse gluons mediating the pairing interaction alter the gap magnitude in a way dependent on the color structure of the gap. We estimate corrections by these effects to the parameters characterizing the fourth order terms in the Ginzburg-Landau free energy and ensure the stability of a color-flavor locked state near T_c.

PACS numbers: 12.38.Mh,26.60.+c

I. INTRODUCTION

It has been noted since the seminal work in Refs. [1, 2] that the quark-quark interaction in the color antitriplet channel is attractive and drives a Cooper pairing instability in quark matter in the limit of high density in which the Fermi energy of the quarks dominates over the one-gluon exchange interaction energy. Because of the relativistic nature of high density quark matter, the color magnetic (transverse) force plays a dominant role in Cooper pairing. This interaction is nonlocal in time just like the electron-phonon interaction in ordinary superconductors [3, 4], but in contrast is long-ranged in the absence of static screening. This long-range nature gives rise to a different dependence of the pairing gap on the QCD coupling constant g from the BCS result [5]. At zero temperature, up to leading order in g, the logarithm of the gap arises from the dynamically screened magnetic force that involves Landau damped virtual gluons in a normal medium. Corrections to the gluon self-energy (polarization function) by the gap do not affect the logarithm of the gap up to subleading order in g at zero temperature [6]. This is because the gluon self-energy is modified by the gap significantly only at gluon energies of $\lesssim T_c$. The influence of such corrections on the gap equation near the transition temperature T_c, however, has yet to be examined.

In this paper we investigate the structure of the gap equation near T_c by including the polarization effects of the color superconducting medium on exchanged gluons. We then estimate corrections thereby induced to the parameters characterizing the fourth order terms in the Ginzburg-Landau free energy of a weakly coupled uniform superfluid of massless three-flavor quarks. These corrections, divided by the weak coupling value, are of order g, in contrast with the case of a short-range pairing interaction in which the corrections generally contain a factor proportional to the ratio of the transition temperature to the Fermi energy. We find that the polarization corrections keep the color-flavor locked phase the most stable just below T_c. Throughout this paper, we consider a system of three-flavor (uds) and three-color (RGB) massless quarks at temperature T and baryon chemical potential μ, and use units $\hbar = c = 1$. We assume that the Fermi momentum is common to all colors and flavors.

II. GAP EQUATION

In this section we address the question of how the gap equation relevant in the weak coupling regime, in which the pairing interaction is induced by one-gluon exchange, is modified by the polarization effects of the color superconducting medium. For this purpose we first consider a $J^P = 0^+$ pairing state that is ud-isoscalar and RG-color antitriplet, since this is one of the simplest states that belong to a color and flavor antisymmetric channel with $J^P = 0^+$. This channel has a common transition temperature in the limit of weak coupling [7]:

$$T_c = \frac{2e^\gamma}{\pi} e^{-(\pi^2+4)/8} \quad \frac{b\mu}{3} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right), \tag{1}$$

where $e^\gamma/\pi = 0.5669\ldots$ and $b = 256\pi^4(2/3g^2)^{5/2}$. For the isoscalar pairing state of interest here, a nonzero excitation gap $d(\mathbf{k})$ is open below T_c for quasiparticles having either flavor u or d and either color R or G. Here $\mathbf{k}$ is the momentum associated with the relative coordinate of a quark Cooper pair. In the case in which only the modification of the one-gluon exchange force by a normal medium is included in the random-phase approximation (RPA) [8, 9]

and the normal state Hartree-Fock correction to the quark propagator is ignored, the gap equation reads [10, 11]

$$\begin{aligned} d(\mathbf{k}) &= \frac{g^2}{48\pi^3}\int d^3q[D_T(E(\mathbf{q})-E(\mathbf{k}),\mathbf{q}-\mathbf{k})+D_T(E(\mathbf{q})+E(\mathbf{k}),\mathbf{q}-\mathbf{k}) \\ &\quad +D_L(E(\mathbf{q})-E(\mathbf{k}),\mathbf{q}-\mathbf{k})+D_L(E(\mathbf{q})+E(\mathbf{k}),\mathbf{q}-\mathbf{k})] \\ &\quad \times d(\mathbf{q})E^{-1}(\mathbf{q})\tanh\left(\frac{E(\mathbf{q})}{2T}\right), \end{aligned} \tag{2}$$

where

$$E(\mathbf{q}) = \left[\left(|\mathbf{q}|-\frac{\mu}{3}\right)^2 + d^2(\mathbf{q})\right]^{1/2} \tag{3}$$

is the excitation energy,

$$D_T(p) \simeq \mathrm{Re}\left[\frac{1}{|\mathbf{p}|^2 - i\pi m_D^2 p_0\theta(\sqrt{\pi}m_D/2-|\mathbf{p}|)/4|\mathbf{p}|}\right] \tag{4}$$

and

$$D_L(p) \simeq \frac{1}{|\mathbf{p}|^2+m_D^2}, \tag{5}$$

with the Debye screening mass

$$m_D = \left[\frac{3g^2}{2\pi^2}\left(\frac{\mu}{3}\right)^2 + \frac{3g^2T^2}{2}\right]^{1/2}, \tag{6}$$

characterize the transverse and longitudinal parts of the gluon propagator $D(p)$ in the Landau gauge as

$$D^{\alpha\beta}_{\mu\nu}(p) = -\delta_{\alpha\beta}[P^T_{\mu\nu}D_T(p)+P^L_{\mu\nu}D_L(p)], \tag{7}$$

with the transverse and longitudinal projection operators

$$P^T_{ij} = \delta_{ij} - \frac{p_ip_j}{|\mathbf{p}|^2}, \quad P^T_{00} = P^T_{0i} = P^T_{i0} = 0, \tag{8}$$

$$P^L_{\mu\nu} = \frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} - P^T_{\mu\nu}. \tag{9}$$

Expressions (4) and (5) are approximate in the sense that they are available in the regime $p_0 \ll |\mathbf{p}| \ll \mu/3$, but duly allow for the Landau damping of transverse virtual gluons and the Debye screening of the longitudinal force in a way sufficient to describe the exact form of the logarithm of the gap magnitude up to subleading order in g [10]. Note that the Landau damping provides an effective infrared cutoff in the transverse sector, $\sim (\pi m_D^2|p_0|/4)^{1/3}$, which in turn plays a dominant role in determining the pairing gap.

At $T=0$, the solution to the gap equation (2) is known as [10]

$$d(\mathbf{k}) = \frac{2}{3}b\mu e^{-\pi/2\bar{g}}\sin(\bar{g}x), \tag{10}$$

where $\bar{g} \equiv g/3\sqrt{2}\pi$, and

$$x \equiv \ln\left[\frac{2b\mu/3}{||\mathbf{k}|-\mu/3|+E(\mathbf{k})}\right]. \tag{11}$$

The factor $\sin(\bar{g}x)$ ensures that the gap is appreciable only for momenta $\mathbf{k}$ close to the Fermi surface. The exponential term and the sinusoidal x dependence arise from nearly static, Landau-damped magnetic gluons that mediate the long-range part of the magnetic interactions, while both the higher frequency magnetic gluons and Debye-screened electric gluons play a dominant role in determining the pre-exponential factor.

For later comparison with the case near T_c, we write down the equation for the magnitude of the gap on the Fermi surface, $d_F \equiv d(|\mathbf{k}| = \mu/3)$, which can be derived from Eq. (2) as [10]

$$d_F = \frac{2g^2}{(3\pi^2)^2}\left[\ln^2\left(\frac{2\delta}{d_F}\right) + b' \ln\left(\frac{2\delta}{d_F}\right)\right] d_F. \tag{12}$$

Here the cutoff δ, obeying $d_F \ll \delta \ll m_D$, is chosen so that $d(|\mathbf{k}| > \delta)$ is vanishingly small, and $b' = 2\ln(b\mu/3\delta)$. The term associated with $\ln^2(2\delta/d_F)$ comes from soft Landau-damped magnetic gluons, while the term associated with $\ln(2\delta/d_F)$ comes from nonstatic magnetic gluons and Debye-screened electric gluons.

The overall coefficient of the μ/g^5 in the pre-exponential factor in Eq. (10) is correct up to a factor of order unity since the quasiparticle wave function renormalization ignored here results in a factor $\exp[-(\pi^2+4)/8]$ [12], which appears also in the weak coupling expression for T_c, Eq. (1). (This renormalization affects the sinusoidal x dependence only through a factor of order $g^2 x$.) On the other hand, polarization by the color superconducting medium, which gives rise to the gap dependence of the pairing interaction, provides even higher order corrections to the zero-temperature gap [6].

Near T_c, the momentum dependence of the gap can be set equal to that at $T = 0$, as in the usual BCS case [10]. Consequently,

$$d(\mathbf{q}, T) = d_F(T)\sin(\bar{g}y), \tag{13}$$

where

$$y \equiv \ln\left[\frac{2b\mu/3}{||\mathbf{q}| - \mu/3| + E(\mathbf{q}, T=0)}\right]. \tag{14}$$

Then, the gap magnitude d_F can be determined by expanding the gap equation (2) up to $\mathcal{O}(d^3)$ as

$$\begin{aligned} d_F \;=\; & \frac{g^2}{18\pi^2}\int_0^\delta d(|\mathbf{q}| - \mu/3)\ln\left(\frac{b\mu/3}{||\mathbf{q}| - \mu/3|}\right) \\ & \times\left\{d(\mathbf{q})\frac{\tanh\left(||\mathbf{q}| - \mu/3|/2T\right)}{||\mathbf{q}| - \mu/3|} + d^3(\mathbf{q})\frac{1}{2||\mathbf{q}| - \mu/3|}\frac{d}{d||\mathbf{q}| - \mu/3|}\left[\frac{\tanh\left(||\mathbf{q}| - \mu/3|/2T\right)}{||\mathbf{q}| - \mu/3|}\right] + \cdots\right\}, \end{aligned} \tag{15}$$

where we have noted that the momentum region, $\mathbf{k} \approx \mathbf{q}$, contributes dominantly to the integral in Eq. (2). We thus obtain

$$d_F = \left(1 - \frac{\pi\bar{g}}{2}\frac{T - T_c}{T_c}\right) d_F - \frac{7\zeta(3)\bar{g}}{16\pi T_c^2} d_F^3 + \mathcal{O}(d_F^5), \tag{16}$$

with the zeta function $\zeta(3) = 1.2020\ldots$. Here the coefficients affixed to d_F and d_F^3 include the leading contributions with respect to g, and we have noted that at $T = T_c$,

$$1 \;=\; \frac{g^2}{18\pi^2}\int_0^\delta d(|\mathbf{q}| - \mu/3)\ln\left(\frac{b\mu/3}{||\mathbf{q}| - \mu/3|}\right)\sin(\bar{g}y)\frac{\tanh\left(||\mathbf{q}| - \mu/3|/2T_c\right)}{||\mathbf{q}| - \mu/3|} \tag{17}$$

is satisfied. [In the absence of the quasiparticle wave function renormalization, the solution to Eq. (17) reproduces expression (1) except for a factor $\exp[-(\pi^2+4)/8]$.] The solution to Eq. (16) reads

$$d_F = \left[\frac{8\pi^2 T_c^2}{7\zeta(3)}\frac{T - T_c}{T_c}\right]^{1/2}. \tag{18}$$

As it should, Eq. (16) is the same as the known result obtained from the Ginzburg-Landau theory (see the next section).

We now introduce the effect of the color superconducting medium on the gluon propagator within the RPA. The normal gluon propagator characterized by Eqs. (4) and (5) is modified by the pairing gap, in a way dependent on α, as [13]

$$D_T^\alpha(p) \simeq \mathrm{Re}\left[\frac{1}{|\mathbf{p}|^2 + (m_M^\alpha)^2 f(\mathbf{p}) - i\pi m_D^2 p_0 \theta(\sqrt{\pi}m_D/2 - |\mathbf{p}|)/4|\mathbf{p}|}\right] \tag{19}$$

and

$$D_L^\alpha(p) \simeq \frac{1}{|\mathbf{p}|^2 + m_D^2 - 3(m_M^\alpha)^2 h(\mathbf{p})}. \tag{20}$$

Here only leading corrections by the pairing gap have been retained, and

$$(m_M^\alpha)^2 = \begin{cases} 0, & \alpha = 1,2,3, \\ g^2 K_T d_F^2, & \alpha = 4,5,6,7, \\ (4/3) g^2 K_T d_F^2, & \alpha = 8, \end{cases} \tag{21}$$

are the Meissner screening masses [14] with the stiffness parameter in the weak coupling limit:

$$K_T = \frac{7\zeta(3)}{24(\pi T_c)^2} N\left(\frac{\mu}{3}\right), \tag{22}$$

where

$$N\left(\frac{\mu}{3}\right) = \frac{1}{2\pi^2}\left(\frac{\mu}{3}\right)^2 \tag{23}$$

is the ideal gas density of states at the Fermi surface. $f(\mathbf{p})$ and $h(\mathbf{p})$ are the dimensionless positive definite functions that characterize the $\mathcal{O}(d^2)$ corrections to the transverse and longitudinal parts of the irreducible particle-hole bubble [13]. These functions reduce to unity in the limit of $\mathbf{p} \to 0$, while decreasing to zero with increasing $|\mathbf{p}|$. We note that the Landau damping term, corresponding to the energy-dependent term in Eq. (19), does undergo corrections by a factor of $1 + \mathcal{O}(d^2/|\mathbf{p}|^2)$, but they lead to higher order corrections to the gap equation in T_c/μ as compared with those coming from the Meissner term $(m_M^\alpha)^2 f(\mathbf{p})$.

Expressions (19) and (20) are the straightforward extension of the normal medium forms (4) and (5) to the case of the color superconducting medium. These expressions for D_T and D_L retain consistency with the transverse and longitudinal sum rules obeyed by the static, long-wavelength gluon propagator in normal quark matter [10] and in color superconducting quark matter [14].

It is important to note that the leading feedback effect of the pairing gap lies in the magnetic sector. This is partly because the corrections to the gluon propagator are roughly of order $m_M^2/(m_D^2|p_0|)^{2/3}$ in the magnetic sector, while of order m_M^2/m_D^2 in the electric sector, and partly because the gluon energy range $|p_0| \ll m_D$ plays a dominant role in the gap equation. In deriving such corrections in the magnetic sector, we first expand D_T, Eq. (19), with respect to m_M^2, and then substitute into $f(\mathbf{p})$ the form relevant near T_c:

$$f(\mathbf{p}) = \frac{6}{7\zeta(3)} \sum_{s=0}^{\infty} \int_0^1 dx \frac{1 - x^2}{(s+1/2)[4(s+1/2)^2 + (|\mathbf{p}|x/2\pi T_c)^2]}. \tag{24}$$

This form is identical to that encountered in the usual BCS case [15] since in both cases a pairing gap is open for quasiparticle momenta so close to the Fermi surface that quasiparticles and quasiholes having momenta $\mathbf{k}_1$ and $\mathbf{k}_2$ with $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{p}$ and $|\mathbf{k}_1| \simeq |\mathbf{k}_2| \simeq \mu/3$ dominate the $\mathcal{O}(d^2)$ corrections to the particle-hole bubble. In the London limit ($|\mathbf{p}| \to 0$), one can set $f \simeq 1$, while in the Pippard limit ($|\mathbf{p}| \to \infty$) f behaves as $\propto 1/|\mathbf{p}|$. As we shall see, the Pippard regime ($|\mathbf{p}| > 2\pi T_c$) is as important to the gap equation as the London regime ($|\mathbf{p}| < 2\pi T_c$).

The gap equation modified by the leading feedback effect can be obtained by replacing D_T by

$$D_T^\alpha(p) \simeq \mathrm{Re}\left[\frac{1}{|\mathbf{p}|^2 - i\pi m_D^2 p_0 \theta(\sqrt{\pi} m_D/2 - |\mathbf{p}|)/4|\mathbf{p}|}\right] - (m_M^\alpha)^2 f(\mathbf{p}) \mathrm{Re}\left\{\frac{1}{[|\mathbf{p}|^2 - i\pi m_D^2 p_0 \theta(\sqrt{\pi} m_D/2 - |\mathbf{p}|)/4|\mathbf{p}|]^2}\right\} \tag{25}$$

in Eq. (2). The modification associated with m_M^α works only for $\alpha = 8$. This is partly because the Meissner masses vanish for $\alpha =$ 1–3 [see Eq. (21)] and partly because the contributions of gluons of $\alpha =$ 4–7 to the gap equation vanish due to the color structure of the gap [10]. Near T_c, the gap equation at $|\mathbf{k}| = \mu/3$ thus reads

$$\begin{aligned} d_F &= \left(1 - \frac{\pi\bar{g}}{2}\frac{T - T_c}{T_c}\right) d_F - \frac{7\zeta(3)\bar{g}}{16\pi T_c^2} d_F^3 \\ &\quad + \frac{g^2}{8\pi^2}\int_0^\delta d(|\mathbf{q}| - \mu/3) d(\mathbf{q}) \frac{\tanh(||\mathbf{q}| - \mu/3|/2T_c)}{||\mathbf{q}| - \mu/3|}\left(\frac{1}{12}\right)(m_M^{\alpha=8})^2 \frac{1}{2|\mathbf{q}|^2} F(\mathbf{q}), \\ &\quad + \mathcal{O}(d_F^5), \end{aligned} \tag{26}$$

where

$$F(\mathbf{q}) = \frac{1}{3}\int_{-1}^{1} d(\cos\theta)\frac{1-\cos\theta}{(1-\cos\theta)^3 + (\pi m_D^2||\mathbf{q}|-\mu/3|/8\sqrt{2}|\mathbf{q}|^3)^2\theta(\sqrt{\pi}m_D/2-\sqrt{2}|\mathbf{q}|\sqrt{1-\cos\theta})}\left[f(y)+y\frac{df(y)}{dy}\right] \tag{27}$$

with $y = |\mathbf{q}|\sqrt{1-\cos\theta}/\sqrt{2}\pi T_c$. Here, θ is the angle between $\mathbf{k}$ and $\mathbf{q}$, we have replaced $|\mathbf{k}|$ by $\mu/3$ in the gluon propagator, and the factor $1/12$ comes from the color vertex part of $\alpha = 8$. Note that the leading term by the feedback effects in Eq. (26) is of third order in d_F and hence does not affect T_c.

We proceed to examine the leading corrections to the gap equation due to the color superconducting medium. For this purpose, it is useful to divide the gluon momentum $|\mathbf{p}| \simeq \sqrt{2}|\mathbf{q}|\sqrt{1-\cos\theta}$ and energy $p_0 \simeq ||\mathbf{q}|-\mu/3|$ into several regimes shown in Fig. 1. The boundary $y=1$ corresponds to the gluon momentum $|\mathbf{p}| = 2\pi T_c$. For $y<1$ (London regime) and $y>1$ (Pippard regime), $f(y)$ has the following forms:

$$f(y) = \sum_{n=0}^{\infty} a_n y^{2n}, \quad a_n = \frac{24}{7\zeta(3)}\frac{(-1)^n\zeta(2n+3)}{(2n+1)(2n+3)}\left[1-2^{-(2n+3)}\right], \tag{28}$$

and

$$f(y) = \frac{6}{7\zeta(3)y}\sum_{s=0}^{\infty}\left\{\left[\frac{1}{2(s+1/2)^2}+\frac{2}{y^2}\right]\tan^{-1}\frac{y}{2(s+1/2)} - \frac{1}{(s+1/2)y}\right\}. \tag{29}$$

At $y=1$, as can be seen from Eq. (27), the effective infrared cutoff due to the Landau damping of transverse gluons is nonnegligible when p_0 is larger than $32\pi^2T_c^3/m_D^2$. Another important scales are $p_0 = \pi T_c$, above which $\tanh(p_0/2T_c)$ in Eq. (26) approaches unity exponentially, and $|\mathbf{p}| = \sqrt{\pi}m_D/2$, above which transverse gluons no longer undergo Landau damping. We note that the gap can be regarded as flat in regions a) and c). We also note that the timelike regime ($p_0 > |\mathbf{p}|$), in which the gluon propagator is not described well by the form (25), can be safely ignored since in this regime no feedback corrections to the gap equation occur up to leading order in T_c/μ. In fact, the gap corrections to the gluon propagator vanish like $\sim d^2/p_0^2$ with increasing p_0, as in the $T=0$ case [6].

At $2\pi T_c < |\mathbf{p}|$, we can use expression (29) for $f(y)$. From this expression, we obtain

$$f(y)+y\frac{df(y)}{dy} = \frac{12}{7\zeta(3)y^2}\mathcal{F}(y), \tag{30}$$

with

$$\mathcal{F}(y) = \sum_{s=0}^{\infty}\left[-\frac{2}{y}\tan^{-1}\frac{y}{2(s+1/2)}+\frac{1}{s+1/2}\right]. \tag{31}$$

The large y asymptotic behavior of $f(y)+ydf(y)/dy$ is then

$$f(y)+y\frac{df(y)}{dy} \approx \frac{12}{7\zeta(3)y^2}\ln y. \tag{32}$$

This behavior is different from $\propto y^{-1}$, which is followed by $f(y)$. As a result, transverse gluons of momenta near T_c rather than near the Pippard limit are essential to calculations of the feedback effect in the magnetic sector. This is a contrast to the case of the weak coupling limit in which transverse gluons of momenta large compared with T_c dominate the pairing interaction since for such momenta, the factor $m_D^2 p_0/|\mathbf{p}|$ characterizing the Landau damping in the propagator (4) is sufficiently small that the pairing interaction remain essentially long ranged.

Using Eqs. (27) and (29), we calculate the contribution to the gap equation (26) from regions c) and d). The momentum range covering these regions corresponds to the range of θ satisfying $-1 < \cos\theta < 1-2\pi^2T_c^2/|\mathbf{q}|^2$. Up to leading order in g, the result from region c) reads

$$\frac{\pi[3\pi^3 - 28\zeta(3)f(1)]\bar{g}^2(m_M^{\alpha=8})^2 d_F}{112\zeta(3)m_D^2}, \tag{33}$$

while the contribution from region d) is of higher order in T_c/μ and thus can be ignored.

At $|\mathbf{p}| < 2\pi T_c$, where expression (28) is available for $f(y)$, $f(y)$ and hence $f(y)+ydf(y)/dy$ are almost flat. In $F(\mathbf{q})$, this momentum range corresponds to the range of θ satisfying $1-2\pi^2T_c^2/|\mathbf{q}|^2 < \cos\theta < 1$. The contribution to the gap equation (26) from region a) becomes, to leading order in g,

$$\frac{\pi\bar{g}^2(m_M^{\alpha=8})^2 d_F}{4m_D^2}\sum_{n=0}^{\infty}a_n, \tag{34}$$

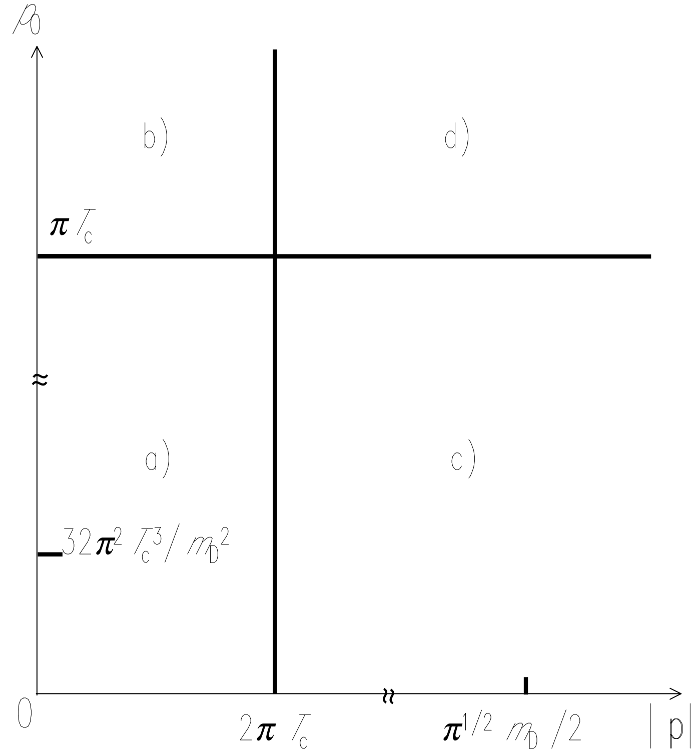

FIG. 1: The energy-momentum regimes of exchanged magnetic gluons.

while that from region b) is of higher order in T_c/μ. It is remarkable that the term (34) is comparable to the term (33). This suggests that transverse gluons of momenta below and above $2\pi T_c$ are equally important to the feedback effect. We also note that these terms are of order $g^2 d_F^3/T_c^2$ and thus suppressed only by one power g with respect to the term proportional d_F^3 in the gap equation (16) in the weak coupling limit. This is a contrast to the case of a short-range pairing force in which the leading correction to the third order term due to the superfluid medium is suppressed by one power T_c/μ [16]. We remark that the scale of the gluon energy dominant in the gap equation is of order $32\pi^2 T_c^3/m_D^2$. Since this is much smaller than the typical momentum scale $\sim 2\pi T_c$, we can safely take the static limit of the modification due to the color superconducting medium.

We finally rewrite the gap equation (26) in such a way that the feedback term has a coefficient up to leading order in g, i.e., by combining the contributions (33) and (34) from regions c) and a). The result is

$$d_F = \left(1 - \frac{\pi\bar{g}}{2}\frac{T-T_c}{T_c}\right) d_F - \frac{7\zeta(3)\bar{g}}{16\pi T_c^2} d_F^3 + \frac{7\zeta(3)C\bar{g}^2}{32\pi T_c^2} d_F^3 + \mathcal{O}(d_F^5), \tag{35}$$

where $C = \pi^3/63\zeta(3) = 0.409434\ldots$. We thus find that the leading feedback effect near T_c acts to increase the gap squared of the isoscalar pairing state by a factor of $(1 - C\bar{g}/2)^{-1}$. This is due to the fact that the feedback effect manifests itself as Meissner screening of the color magnetic force of color index $\alpha = 8$; this force is repulsive in contrast to the attractive case of $\alpha = 1$–3 dominating the pairing interaction. We remark that in the $T = 0$ case in which expansion of the gap equation with respect to the gap magnitude is not valid, the terms associated with the logarithm of the gap in the gap equation (12) mainly determine the gap magnitude. In this case, the feedback effect provides corrections beyond these logarithmic terms [6].

For the purpose of calculating corrections to the Ginzburg-Landau parameters in the next section, it is instructive to

repeat the above calculations for a color-flavor locked (CFL) state, one of the $J^P = 0^+$, color and flavor antisymmetric pairing states. In the CFL state, all three flavors and colors are equally gapped in such a way that the pairing gap between a quark of color a and flavor i and a quark of color b and flavor j is characterized by $\kappa(\delta_{ai}\delta_{bj} - \delta_{aj}\delta_{bi})$. For the on-shell gap on the Fermi surface, κ_F, the gap equation near T_c can be written in the form similar to Eq. (35) as

$$\kappa_F = \left(1 - \frac{\pi\bar{g}}{2}\frac{T - T_c}{T_c}\right)\kappa_F - \frac{7\zeta(3)\bar{g}}{8\pi T_c^2}\kappa_F^3 - \frac{21\zeta(3)C\bar{g}^2}{8\pi T_c^2}\kappa_F^3 + \mathcal{O}(\kappa_F^5). \tag{36}$$

Here we have used the Meissner masses in the CFL state [14], i.e., $(m_M^\alpha)^2 = 2g^2 K_T \kappa_F^2$ for $\alpha = 1$–8. In the limit of $m_M^\alpha \to 0$, Eq. (36) is equivalent to Eq. (104) in Ref. [11]. We thus find that the leading feedback effect acts to reduce the gap squared by a factor of $(1 + 3C\bar{g})^{-1}$. This reduction stems from the fact that Meissner screening of the color magnetic force takes effect equally for $\alpha = 1$–8. Note a contrast with the case of the isoscalar pairing state in which the feedback effect acts to increase the gap magnitude.

III. GINZBURG-LANDAU FREE ENERGY

We proceed to derive the Ginzburg-Landau free energy of a weakly coupled uniform superfluid of massless three-flavor quarks from the gap equation near T_c as examined in the previous section. Instead of focusing on the isoscalar and CFL pairing states, it is convenient to construct the Ginzburg-Landau free energy for a more general color and flavor antisymmetric channel with $J^P = 0^+$ as in Ref. [11]. This is because all states belonging to this channel has a common value of T_c, which reduces to Eq. (1) in the weak coupling limit. This channel is characterized by a complex 3×3 gap matrix, $(\mathbf{d}_a)_i$, in color-flavor space [11], where a (i) is the color (flavor) other than two colors (flavors) involved in Cooper pairing. This gap is defined on the mass shell of the quark quasiparticle of momenta on the Fermi surface, and thus reduces to $\delta_{aB}\delta_{is}d_F$ in the isoscalar state and to $\delta_{ai}\kappa_F$ in the CFL state. For $(\mathbf{d}_a)_i$, one can write down the thermodynamic potential density difference $\Delta\Omega = \Omega_s - \Omega_n$ between the superfluid and normal phases near T_c as [11]

$$\Delta\Omega = \bar{\alpha}\sum_a |\mathbf{d}_a|^2 + \beta_1(\sum_a |\mathbf{d}_a|^2)^2 + \beta_2\sum_{ab}|\mathbf{d}_a^* \cdot \mathbf{d}_b|^2. \tag{37}$$

Here each term is invariant with respect to $U(1)$ global gauge transformations and color and flavor rotations.

In evaluating the coefficients in Eq. (37) by including the leading feedback effect, it is convenient to integrate the gap equations (35) and (36) and then map the results onto Eq. (37). We thus obtain

$$\bar{\alpha} = 4N(\mu/3)\ln\left(\frac{T}{T_c}\right), \tag{38}$$

$$\beta_1 = \frac{7\zeta(3)}{8(\pi T_c)^2}\left(1 + \frac{13}{2}C\bar{g}\right)N(\mu/3), \tag{39}$$

$$\beta_2 = \frac{7\zeta(3)}{8(\pi T_c)^2}\left(1 - \frac{15}{2}C\bar{g}\right)N(\mu/3), \tag{40}$$

which reproduce the known relation $\beta_1 = \beta_2$ in the weak coupling limit [11]. We find that up to leading order in g, the polarization effects of the color superconducting medium give rise to only $\mathcal{O}(g)$ corrections of the coefficients β_1 and β_2. It is nonetheless important to note that these corrections to β_1 and β_2 work in the opposite directions in such a way as to decrease and increase the gap magnitude in the CFL and isoscalar states, respectively. We can thus conclude that whether or not the leading feedback effect acts to reduce the gap magnitude depends on the color structure of the gap.

IV. PHASE DIAGRAM

We turn to the construction of the phase diagram in the space of the parameters characterizing the fourth order terms in the Ginzburg-Landau free energy derived in the previous section. This phase diagram can be obtained by minimizing the thermodynamic potential difference $\Delta\Omega$ with respect to $(\mathbf{d}_a)_i$ for various values of β_1 and β_2 [11]. The result, exhibited in Fig. 2, is the same as Fig. 1 in Ref. [11] except that the present values of β_1 and β_2 given by Eqs. (39) and (40) include the $\mathcal{O}(g)$ feedback corrections. This figure shows that even for such values of β_1 and β_2, as long as $\bar{g}$ is sufficiently smaller than unity, the CFL phase, which generally satisfies

$$\mathbf{d}_R^* \cdot \mathbf{d}_G = \mathbf{d}_G^* \cdot \mathbf{d}_B = \mathbf{d}_B^* \cdot \mathbf{d}_R = 0, \quad |\mathbf{d}_R|^2 = |\mathbf{d}_G|^2 = |\mathbf{d}_B|^2, \tag{41}$$

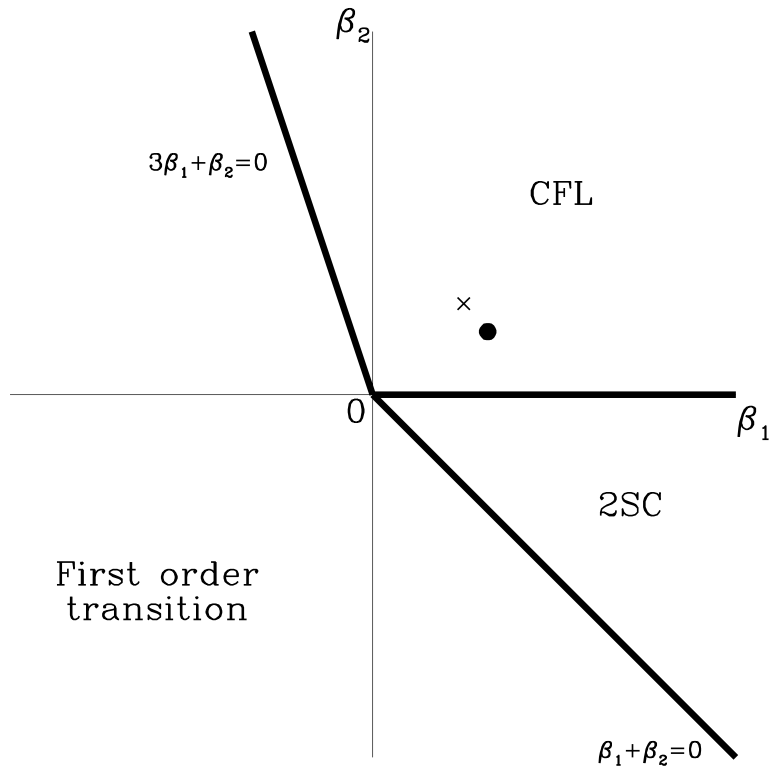

FIG. 2: Phase diagram near T_c, exhibiting regions where the 2SC and CFL phases are reached by a second order transition as well as where superfluid states are reached by a first order transition since the overall fourth order term in the Ginzburg-Landau free energy (37) can be negative. The parameters β_1 and β_2 are the fourth order coefficients in the Ginzburg-Landau free energy. The cross denotes the weak coupling limit, and the circle denotes the result including the polarization effects of the color superconducting medium with $\bar{g} = 0.1$.

is still favored over the two-flavor color superconducting (2SC) state fulfilling

$$\mathbf{d}_R \parallel \mathbf{d}_G \parallel \mathbf{d}_B. \tag{42}$$

Note that the 2SC state contains the isoscalar state analyzed in Sec. II.

V. CONCLUSIONS

We have examined the role played by the gap dependence of the pairing interaction in the gap equation for a weakly coupled uniform superfluid of three-flavor massless quarks near the transition temperature T_c. The corrections induced by this role to the parameters characterizing the fourth order terms in the Ginzburg-Landau free energy result in an increase of β_1 by a factor of $1 + 13C\bar{g}/2$ and an decrease of β_2 by a factor of $1 - 15C\bar{g}/2$. The magnitude of these changes comes from the Meissner screening of the color magnetic force that dominates the interaction between quarks

in the weak coupling regime as an essentially long-range force. We thus see a contrast with the case of a short-range pairing interaction in which the corrections are suppressed by one power T_c/μ. We also note that the changes in β_1 and β_2 are in the direction of increasing the gap magnitude of the 2SC state and decreasing that of the CFL state. This direction reflects the fact that not only the Meissner screening of the color magnetic force but also the color indices of magnetic gluons dominating the pairing interaction depend on the color structure of the pairing gap. We have finally found that the feedback corrections, as long as $\bar{g} \ll 1$, keep the color-flavor locked phase the most stable just below T_c.

The present result for the parameters β_1 and β_2, mainly through its effect on the gap magnitude near T_c, provides a way of studying strong coupling modifications on the previous weak coupling calculations based on the Ginzburg-Landau theory with $\beta_1 = \beta_2$. Those calculations include the phase diagram [11] as discussed in Sec. IV and its extension to nonzero quark masses [17], responses to rotation and magnetic fields [18, 19, 20], and fluctuation-induced first order transition [21]. Qualitatively, however, no significant changes are expected. Nonetheless, the tendency that $\beta_1 > \beta_2$, if remaining at low densities, could be significant for the normal-super interfacial energy and the interaction between widely separated magnetic vortices for CFL quark matter. This is because both quantities, which are sensitive to the ratio between β_1 and β_2 [19, 20], control the criterion of whether or not the CFL state can allow magnetic vortices to form.

Acknowledgments

We are grateful to Gordon Baym, Tetsuo Hatsuda, Taeko Matsuura, and Motoi Tachibana for helpful discussions. We acknowledge the hospitality of the Institute for Nuclear Theory at the University of Washington, where this work was initiated.

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